RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. FIFTH SEMESTER EXAMINATION, DECEMBER 2016

THIRD YEAR [BATCH 2014-17]

Date : 14/12/2016 **PHYSICS [Honours]**

Time : 11 am – 1 pm **Paper :** V [Gr-A&B] Full Marks : 50

Group – A

(***Answer <u>any three</u> questions***) [3×10]

1. a) State with reasons the nature of the constraint in the following :
   i) Pendulum with the point of suspension moving along a curve of given equation
   ii) particle moving inside a room
   iii) particle constrained to move on a surface $f(x, y, z, t) = 0$
   iv) disc rolling without slipping on a plane. [4]

   b) State the principle of virtual work for a constrained N-particle system. Use it to deduce D'Alembert's principle, and explain its physical significance.
   Investigate the motion under gravity of two unequal masses connected by a massless string passing over a massless pulley, using D'Alembert's principle. [4+2]

2. a) Write the Lagrangian of a particle moving with respect to a frame rotating relative to an inertial frame and deduce the equation of motion. [5]

   b) Deduce the conservation principles arising out of the translation and the rotation symmetry of the Lagrangian.
   Is the Lagrangian $L(\vec{r}, \vec{v})$ given by $L(\vec{r}, \vec{v}) = \vec{r}^2 + \vec{r}.\vec{v} + v^2$ rotationally symmetric? [4+1]

3. a) Write the Lagrangian of a particle moving under a central force. Identify the conserved quantities, if any.
   Deduce the Hamiltonian and hence obtain Hamilton's equation of motion. [2+4]

   b) The Hamiltonian H of a particle in a magnetic field is given by $H = \frac{1}{2m}\left(\vec{p} - \frac{e\bar{A}}{c}\right)^2 + e\phi$, where $\bar{A}$ and ϕ are the vector and the scalar potential respectively. Write $\vec{p}$ in terms of $\bar{v}$ from Hamiltan's equation of motion and hence the Lagrangian. [4]

4. a) The Lagrangian $L = \alpha q^2 - \beta \cos q$, $\alpha, \beta > 0$. Find out the stable and the unstable equilibrium points. [3]

   b) The Lagrangian of a two-dimensional motion is given by $L = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2) - \frac{1}{2}\mu(x^2 + y^2) + \frac{\mu}{4}xy$, m and μ being constants. Find out the normal modes corresponding to this Lagrangian. [7]

5. a) Starting from Euler's equation deduce the motion of a rigid body for force-free condition. [5]

   b) i) If the Hamiltonian $H = \alpha q^2 + \beta p^2$, find out q as a function of time using the Poisson bracket formulation,
   [Hint : Use $\frac{dA}{dt} = \frac{\partial A}{\partial t} + [A, H]$] [4]

   ii) If u(q, p) and v(q, p) are constants of motion, show that their Poisson Bracket [u,v] is also a constant of motion. [1]

Group – B

(***Answer any two questions***) [2×10]

6. a) An object moves at velocity v_1 with respect to frame s' and s' moves with velocity v_2 with respect to frame s in the same x direction. Assuming the relativistic velocity addition rule, show that the velocity of the object with respect to s can be given by

$\beta = \tan h(\phi_1 + \phi_2)$ where ϕ_1 and ϕ_2 are the rapidities and are given be $\tan h\phi_1 = \beta_1 = {}^{v_1}\!/_{c}$ and $\tan h\phi_2 = \beta_2 = {}^{v_2}\!/_{c}$. Show that the Lorentz transformation in the x – t plane is given by $\begin{pmatrix} x' \\ ct' \end{pmatrix} = \begin{pmatrix} \cosh\phi & -\sinh\phi \\ -\sinh\phi & \cosh\phi \end{pmatrix}\begin{pmatrix} x \\ ct \end{pmatrix}$.

Where the symbols have the usual meaning. [5]

b) Show that the square of the interval between two space time events is Lorentz-invariant. Distinguish between space-like, time-like and null intervals. Use the invariance of the squared interval to derive the time dilation formula for a moving clock. [3+2]

7. a) Two trains A and B each with proper length L move in the same direction. A's velocity is $\frac{4}{5}c$ and B's velocity is $\frac{3}{5}c$ as observed from the ground. What is the time difference from the ground between the events of front of A passing the back of B, and the back of A passing the front of B. [4]

b) Assume the relativistic momentum of a particle moving with a velocity $\bar{u}$ be $\gamma m\bar{u}$ where m is the rest mass of the particle and $\gamma = \frac{1}{\sqrt{1 - {}^{u^2}\!/_{c^2}}}$. Show that the kinetic energy is given by $k = (\gamma - 1)mc^2$. Hence show that the total energy E is given by $E^2 = c^2p^2 + m^2c^4$ where E and p are the energy and momentum of the particle. Show that in the limit $u \ll c$, the kinetic energy has the correct Newtonian form. [6]

8. a) A rod of length 'l' is lying along the y axis of s frame. If the rod starts moving along y direction with a velocity u, what will be the length of the rod and its velocity as observed from s' moving with velocity v along x direction with respect to the frame s. Find the transformation equations. [4]

b) A train with proper length L moves at velocity $\frac{5}{13}c$ with respect to the ground. A ball is thrown from the back of the train to the front. The velocity of the ball with respect to the train is ${}^{c}\!/_{3}$. As viewed by someone on the ground, how much time does the ball spend in the air and how far does it travel before hitting the front? [3]

c) A nucleus of mass M emits a photon of frequency ν. Show that the nucleus recoils with an energy given by $\frac{h\nu}{2MC^2}$ times the energy of the emitted photon. [3]

9. a) Assuming $\left(\frac{E}{c}, \bar{p}\right)$ transforms as a 4-vector under Lorentz transformation, show by explicit calculation that ${}^{E}\!/_{c}$ transforms as the time component of the space-time 4-vector. Find the invariant 'length' of the energy-momentum 4-vector. Is it a time like or a space like 4-vector for a massive particle? [5]

b) A photon of frequency ν' is emitted by a source at rest in a frame s' at an angle θ' with the x'-axis. s' moves uniformly with respect to an inertial frame s with velocity V along the common x-axis. Show using the energy and momentum transformation equations, that the frequency ν and the angle of emission θ as observed in s are given by

i) $$\nu = \gamma_0 \nu' \left(1 + \frac{V}{C}\cos\theta'\right)$$

and ii) $$\cos\theta = \frac{\cos\theta' + V/C}{1 + V/C \cos\theta'}$$

Hence, discuss the longitudinal and transverse Doppler effects for light. [3+2]

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